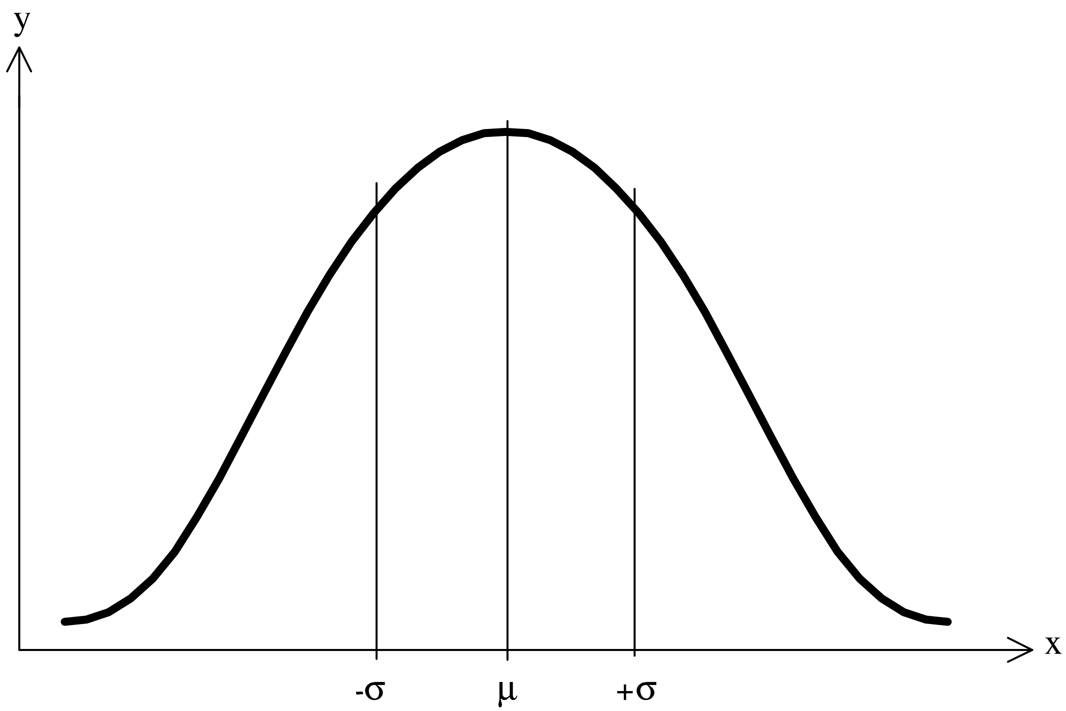
# The Gaussian or Normal Distribution




*The 'Normal' distribution is a mathematically derived 'bell-shaped' curve which many natural processes approximate.*

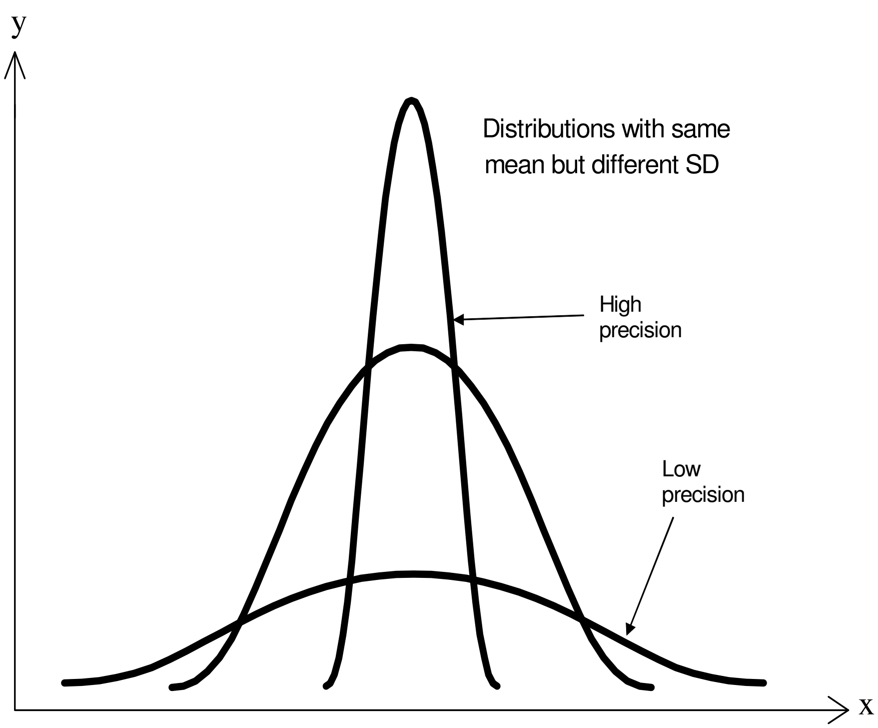
*The equation which defines this curve is as follows ....*

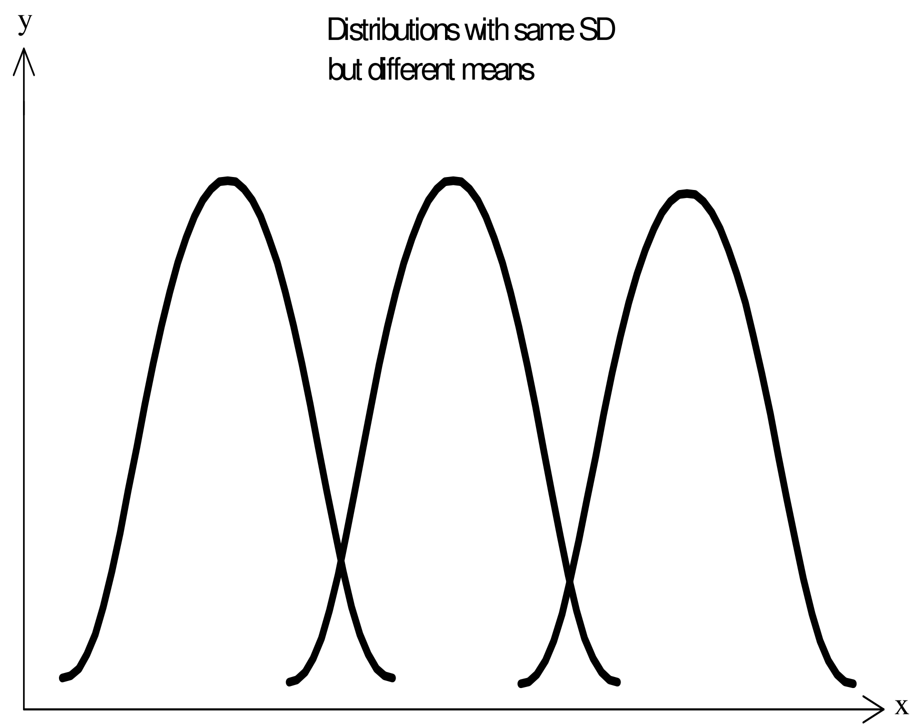
$$y = \frac{1}{\sigma\sqrt{(2\pi)}}e^{-(x-\mu)^2/2\sigma^2}$$

*Two very important parameters are associated with the curve :- the location on the x-axis (indicated by the POPULATION MEAN, $\mu$ or mu) and the process spread (indicated by the STANDARD DEVIATION, $\sigma$ or sigma).*

*These two parameters will be explored in greater depth later.*

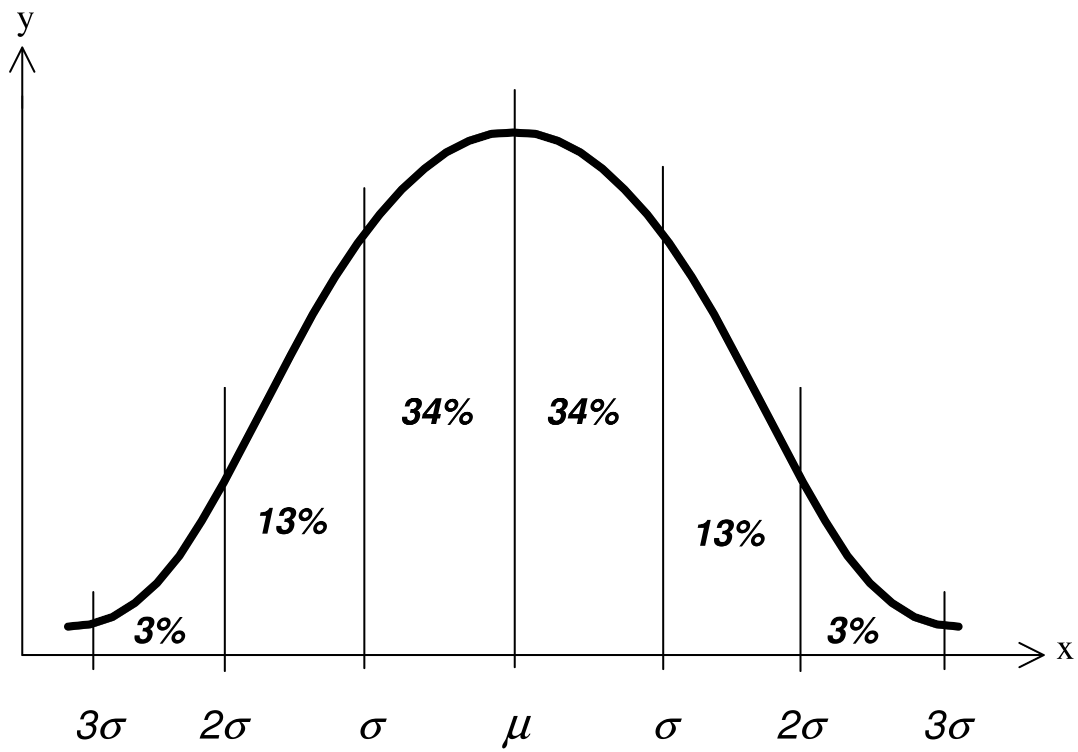
# Properties of Distributions



# The Concept of Standard Deviation

*Whilst the mean gives an indication of central tendency it would also be useful to have some idea of the spread of a population.*




*The population STANDARD DEVIATION is a measure of this spread and is denoted by Greek $\sigma$ (sigma).*

*With reference to the figure left it can be seen that nearly all data values (99.7% to be precise) fall within $\pm 3\sigma$ of the population mean $\mu$ (mu).*

*If a process follows the normal distribution then the $\pm 3\sigma$ lines can be used as CONTROL LIMITS.*

# The Calculation of Standard Deviation

*The STANDARD DEVIATION of a sample is calculated as follows ....*

$$s = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}}$$

*where n is the sample size*
$x_i$ *are the sample values*
$\bar{x}$ *is the sample mean*

*Alternatively the simpler equivalent may be used as follows ....*

$$s = \sqrt{\frac{\sum x_i^{\,2} - n\bar{x}^2}{n-1}}$$

*Once **s** has been calculated it can be used as an estimate of the population standard deviation* $\sigma$.

# Standard Deviation Exercise

*The table below shows the data for the first ten resistors used earlier.*

| | | | | |
|---|---|---|---|---|
| 15.97 | 15.89 | 16.00 | 15.93 | 15.91 |
| 15.95 | 15.91 | 15.85 | 16.01 | 16.09 |

*Using the table below calculate the standard deviation for this sample (Note - you should aim to work to at least 5 significant digits in order to maintain accuracy).*

| *i* | *1* | *2* | *3* | *4* | *5* | *6* | *7* |
|---|---|---|---|---|---|---|---|
| $x_i$ | | | | | | | |
| $x_i^2$ | | | | | | | |

| *8* | *9* | *10* |
|---|---|---|
| | | |
| | | |

| | | | | | |
|---|---|---|---|---|---|
| *n-1* | | $\bar{x}$ | | $\sum x_i^2 - n\bar{x}^2$ | |
| $\sum x_i$ | | $\bar{x}^2$ | | $\frac{\sum x_i^2 - n\bar{x}^2}{n-1}$ | |
| $\sum x_i^2$ | | $n\bar{x}^2$ | | $\sqrt{\frac{\sum x_i^2 - n\bar{x}^2}{n-1}}$ | |
| | | | | | **s** |

# Standard Deviation Exercise - Completed

*The table below shows the data for the first ten resistors used earlier.*

| | | | | |
|---|---|---|---|---|
| 15.97 | 15.89 | 16.00 | 15.93 | 15.91 |
| 15.95 | 15.91 | 15.85 | 16.01 | 16.09 |

*Using the table below calculate the standard deviation for this sample (Note - you should aim to work to at least 5 significant digits in order to maintain accuracy).*

| *i* | *1* | *2* | *3* | *4* | *5* | *6* | *7* |
|---|---|---|---|---|---|---|---|
| $x_i$ | *15.97* | *15.89* | *16.00* | *15.93* | *15.91* | *15.95* | *15.91* |
| $x_i^2$ | *255.0409* | *252.4921* | *256.0000* | *253.7649* | *253.1281* | *254.4025* | *253.1281* |

| | | |
|---|---|---|
| *8* | *9* | *10* |
| *15.85* | *16.01* | *16.09* |
| *251.2225* | *256.3201* | *258.8881* |

| | | | | | |
|---|---|---|---|---|---|
| *n-1* | *9* | $\bar{x}$ | *15.951* | $\sum x_i^2 - n\bar{x}^2$ | *0.0433* |
| $\sum x_i$ | *159.51* | $\bar{x}^2$ | *254.4344* | $\frac{\sum x_i^2 - n\bar{x}^2}{n-1}$ | *0.0048111* |
| $\sum x_i^2$ | *2544.3873* | $n\bar{x}^2$ | *2544.344* | $\sqrt{\frac{\sum x_i^2 - n\bar{x}^2}{n-1}}$ | *0.06936* |
| | | | | | *s* |

***NB** : the SD for all 30 resistors is 0.06509, which agrees quit well with our figure!*